PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The Sources of Lithospheric Tectonic Stresses [and Discussion]

Luce Fleitout, P. England and N. Kusznir

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The sources of lithospheric tectonic stresses

By Luce Fleitout
Laboratoire de Géologie-ENS, 24 rue Lhomond, 75005 Paris, France

The stresses associated with large-scale tectonic deformation have three possible origins: (1) plate-boundary forces counterbalanced by viscous drag beneath the plates; (2) density heterogeneities situated within the plates (say at depths shallower than 200 km); (3) mass heterogeneities in the deep mantle. The first two are shown to be equally important for the understanding of the stress field. No topography (no vertical stress) seems to be associated with lower-mantle mass anomalies. This is most compatible with a two-layer convective mantle where the lower-mantle mass anomalies, mechanically decoupled from the lithosphere, are unable to induce tectonic stresses.

1. Introduction

Large-scale tectonic stresses were first interpreted in terms of plate-boundary forces (ridge push, slab pull, etc.), counterbalanced by the viscous drag beneath the plates. Intralithospheric mass heterogeneities modify the vertical stresses in the lithosphere and are thus an important element of the tectonic stress field (Artyushkov 1973; Fleitout & Froidevaux 1982). Deeper mass heterogeneities in the convective mantle are also potentially important ingredients of the stress field. The purpose of this paper is to discuss the relative importance of these various sources for the stress field.

This paper deals only with 'tectonic stresses', those able to induce large-scale tectonic deformation, i.e. stresses which are not released by a small (less than 1 %) plastic deformation, such as flexural or thermal stresses. The voluntarily simplified models developed here best apply to relatively large-scale tectonic deformation and do not take into account the perturbations which can arise in a three-dimensional geometry from laterally variable mechanical properties or boundary conditions.

2. Density anomalies in the lithosphere and tectonic stresses

(a) *The 'moment' law*

Let us consider a thin mass anomaly Δm, situated in the lithosphere at a depth d. How does it influence the stress field? The exact solutions for a lithosphere made of viscous layers are presented by Fleitout & Froidevaux (1982). Here we present a simple explanation for the results obtained for 'large wavelengths', i.e. in cases where the lateral extent of the mass anomaly $\mathrm{d}x$ is large with respect to the thickness of the lithosphere L. Then, the horizontal strain rate ϵ_{xx} is independent of z (the difference between σ_{xx} and σ_{zz} varies as a function of depth so that ϵ_{xx} remains constant). The strain rate is then only a function of the integral

$$\Sigma = \int_0^L (\sigma_{xx} - \sigma_{zz})\,\mathrm{d}z. \tag{1}$$

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The horizontal equilibrium of a plate section is written as

$$\frac{\partial}{\partial x}\int_0^L \sigma_{xx}\,\mathrm{d}z = -\sigma_{xzL}. \tag{2}$$

Equation (2) implies that over short distances, $\int_0^L \sigma_{xx}\,\mathrm{d}z$ can be considered constant. Variations of the tectonic style between nearby areas only arise from variations of the averaged vertical stress.

How is the vertical stress σ_{zz} affected by the mass anomaly? Let us examine in figure 1 the difference between the vertical stresses over column B, away from the mass anomaly δm and a column containing the mass anomaly (column A).

At the depth L, σ_{zz} is the same for the two columns (similar pressure at a given depth in the fluid sublithospheric mantle). The vertical momentum equation is written as

$$\partial\sigma_{zz}/\partial z + \partial\sigma_{xz}/\partial x = -\Delta\rho\, g, \tag{3}$$

where $\Delta\rho$ is the local density anomaly. (We note that extensional stresses are positive.)

When the depth of the mass anomaly is small compared with its lateral extent $\partial\sigma_{xz}/\partial x$ can be neglected (see Fleitout & Froidevaux (1982) for cases where this condition is not verified). Then, (3) reduces to

$$\partial\sigma_{zz}/\partial z = -\Delta\rho\, g.$$

$\partial\sigma_{zz}/\partial z$ is then the same for the two columns, except for $z = d$:

$$\partial(\sigma_{zz\mathrm{A}} - \sigma_{zz\mathrm{B}})/\partial z = 0 \quad \text{for} \quad z \neq d. \tag{4}$$

$\sigma_{zz\mathrm{A}} - \sigma_{zz\mathrm{B}}$ jumps by $\delta m g$ in $z = d$. $\sigma_{zz\mathrm{A}} - \sigma_{zz\mathrm{B}}$ as a function of depth is presented on figure 1. It follows:

$$\int_0^L (\sigma_{zz\mathrm{A}} - \sigma_{zz\mathrm{B}})\,\mathrm{d}z = \delta m g d. \tag{5}$$

The influence of an intralithospheric mass anomaly on the stress field is therefore proportional to its moment, which is the product of its amplitude and its depth. (This corresponds indeed to the 'moment' of the mass dipole formed by the mass anomaly at depth and the topography that it compensates.) A positive mass anomaly induces tectonic compression ($\sigma_{zz} > \sigma_{xx}$) and, vice versa, a negative mass anomaly induces extension.

Relationship (5) can be generalized to the case of a density distribution that is a function of the depth z:

$$\int_0^L \Delta\sigma_{zz}\,\mathrm{d}z = g\int_0^L \Delta\rho(z)\, z\,\mathrm{d}z. \tag{6}$$

Note that according to isostasy the topography is written as

$$h = -\int_0^L \Delta\rho(z)\,\mathrm{d}z/\rho_\mathrm{c}, \tag{7}$$

where ρ_c is the density of the crust. The two integrals in (6) and (7) do not necessarily have opposite signs (i.e. positive topography does not necessarily correspond to extension and vice versa). Such is the case in figure 2, where the shallow mass

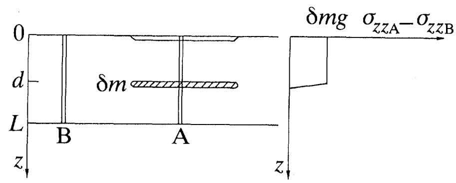

Figure 1. The mass anomaly δm, located at a depth d, induces a vertical stress increase δmg between d and the Earth's surface. $\int_0^L \sigma_{zz}\,\mathrm{d}z$ is in turn increased by δmgd.

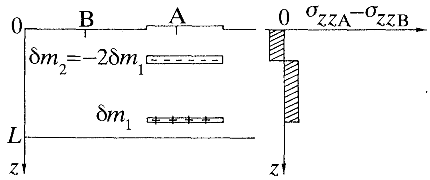

Figure 2. Despite the fact that the relief is high around A, tectonic 'compression' is stronger in A than in B. The 'moment' of the positive deep mass anomaly δm_1 is larger than the 'moment' of the negative shallower mass anomaly δm_2. Consequently, $\int_0^L (\sigma_{zz\mathrm{A}}-\sigma_{zz\mathrm{B}})\,\mathrm{d}z$ (which corresponds to the difference between the shaded areas on the right of the figure) is positive.

anomaly of amplitude $-2\Delta m_1$ (Δm_1 is positive), has a dominant effect on the topography (i.e. the topography is positive), but the positive mass anomaly Δm_1, which is three times deeper, imposes a compressive tectonic style. The effect of intralithospheric mass anomalies cannot therefore be directly linked with the topography. Indeed in the next paragraph we see examples of high topography mountain chains which are in compression with respect to the surrounding, lower-altitude areas.

Note that the 'moment law' (5) is only valid for large wavelengths (i.e. when the lateral extent of the mass anomaly is large with respect to the thickness of the lithosphere). Various examples of the breakdown of this law at short wavelengths, based on the resolution of the Navier–Stokes equation, can be found in Fleitout & Froidevaux (1982).

(*b*) *Some geological examples*

High plateaus: a bound on the magnitude of deviatoric stresses in the lithosphere. The recent tectonics of both Tibet and the Andes indicate that, when a plateau reaches an altitude of about 3000 m, the tectonic style becomes extensional. The vertical stress integrated over the whole lithospheric thickness becomes smaller (more compressive) than the horizontal stress. The excess of integrated vertical stress linked with the plateau topography and its compensation is at most of the order of 6×10^{12} Pa m (Froidevaux & Isacks 1984), which means that the integrated deviatoric stress in the moderate altitude areas surrounding the plateaus is also of the order of 6×10^{12} Pa m. Because of the geometry of continental collision and subduction in the convergence between India and Asia, the Tibet area is expected to experience one of the largest horizontal compressions in the world. The above considerations (extension in Tibet) imply that, even there, the integrated stress does not exceed 6×10^{12} Pa m. This is an important constraint on the magnitude of deviatoric stresses in the plates. Subduction in the Andes may somewhat complicate the picture.

Mountain chains in compression: a consequence of subduction of the continental lithosphere.

Plateaus correspond to the straightforward cases where high-altitude areas are in extension. On the other hand, collisional mountain chains correspond to high-altitude areas in compression, while the lower-altitude surrounding areas are often in extension. For example deformation since the Oligocene in the Massif Central has been characterized by E–W extension with sometimes N–S compression. The N–S Limagne and Forez grabens are the most prominent tectonic features there. Just east of the Massif Central, the French Alps is a compressional mountain chain undergoing E–W compression (Brereton & Müller 1991). The altitude is lower in the Massif Central than in the Alps. This is a case similar to that of figure 2; as indicated by the topography, the integrated density anomalies are more negative below the Alps than below the Massif Central. The moment of these negative densities is, however, larger for the Massif Central. Indeed the high altitude in the Massif Central is not due to crustal thickening, but to a warming up of the lithosphere and to a deep negative density anomaly with a large moment. In the Alps the altitude is due to crustal thickening, most certainly accompanied by a thickening of the lithosphere (Babuska *et al*. 1984); as in figure 2, there is a mass anomaly at shallow depth which is negative below the Alps and positive below the Massif Central. At large depths however, the mass anomalies are positive under the Alps and negative under the Massif Central. This explains the difference of tectonic style between the two areas. Note that in Europe there are numerous similar cases of compressional mountain chains accompanied by nearby extensional basins (Tuscany and Appennines, Pannonian Basin and Carpathian mountains, etc.).

Even though in nature the geometry of deformation is often more complicated, a uniform and adiabatic thickening or thinning of the lithosphere can be considered as a simplified model of these tectonic processes. It can be shown (Fleitout & Froidevaux 1982) that for standard values for a young continental platform (30 km thick crust, 100 km thick thermal lithosphere), thickened lithospheres tend to be more in compression than normal lithospheres, and thinned lithospheres are more in extension. This simply results from the fact that, if one compares a column of standard lithosphere to a column of hot mantle, the moment of the shallow negative mass anomalies due to crustal thickening is smaller (in absolute value) than the moment of the cold-temperature anomaly at depth. These two quantities may become equal for thinner lithospheres, implying also thinner or denser crusts. The moment due to the cold mass anomalies at depth is, however, rarely negligible compared with that of the crust. Simple isostatic comparison between a ridge and a continent indicate that the mantle below a continent, with an altitude close to zero and a crust 30 km thick (or thicker), is necessarily denser than the mantle below a ridge. Models of subsidence of continental basins indicate, moreover, that the continental lithosphere tends to cool and become denser as a function of time in the same manner as the oceanic lithosphere. There is therefore no reason, *a priori* to assume either the absence of lithosphere or a neutrally buoyant lithosphere below the continental crust. Large-scale models of tectonic deformation then need to deal with the not yet well-understood problem of the mechanical behaviour and stability of the thickened cold lithospheric root.

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(c) *How to introduce 'the moment law' in a plane-stress finite-element program?*

A horizontal volume force, proportional to the horizontal gradient of the moment, just needs to be incorporated in the program. Fleitout & Froidevaux (1983) gave the equations for the case of spherical geometry. In the case of plane geometry the equations are written

$$L\left(\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}\right)=\sigma^L_{xz}\frac{-\partial M}{\partial x},\quad L\left(\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{xy}}{\partial x}\right)=\sigma^L_{yz}\frac{-\partial M}{\partial y},$$

where x and y are the two horizontal directions, σ^L_{xz} and σ^L_{yz} are the resistive shear stresses due to the mantle drag below the plate and M is the moment of the lithospheric mass anomalies. Indeed, this approach has often been applied to cases where the moment is simply due to crustal thickening.

3. The relative magnitude of stresses due to 'forces acting on the plates' and to intralithospheric mass anomalies

Stresses due to intralithospheric mass anomalies are sometimes considered as a second-order effect compared with 'forces applied to the plates'. We want to emphasize here that this is not our opinion; the forces applied to the plates are of the same order of magnitude as the ridge push which is a consequence of intralithospheric mass anomalies.

(a) *The ridge push: a variation of the vertical stress due to the increase of density in the lithosphere with age*

The 'ridge push' is well known as a force applied to the plates and is written

$$F_{\mathrm{rp}}=\int_0^L\Delta\rho\, z\,\mathrm{d}z,$$

where $\Delta\rho$ corresponds to the density difference between an old ocean and a young ocean. It can also be written as $F_{\mathrm{rp}}=\int_0^L\Delta\sigma_{zz}\,\mathrm{d}z$. It corresponds to the difference in the integrated vertical stress due to the cooling of the lithosphere. It is evaluated to be about 2×10^{12} Pa m (Dahlen 1981; Fleitout & Froidevaux 1983).

The 'tectonic stress' in the oceanic plate depends upon the variation of $\int_0^L(\sigma_{xx}-\sigma_{zz})\,\mathrm{d}z$ as a function of age. The horizontal variation of σ_{xx} away from the ridge is given by the horizontal equilibrium

$$\frac{\partial}{\partial x}\int_0^L\sigma_{xx}\,\mathrm{d}z=-\sigma_{xz},\tag{2}$$

where σ_{xz} is the shear stress beneath the plate. What is the sign and amplitude of σ_{xz}? In §4, it is argued that σ_{xz} is resisting the plate motion (in other words, we think passive drag to be more likely than active shear below the plates). σ_{xz} will then tend to induce an increase of tectonic extension away from the ridge. Data concerning the state of stress in the oceans indicate that the tectonic style becomes compressive at a short distance from the ridge (Stein & Pelayo 1991). This indicates that the shear stress below the plate is not large enough to overcome the compression induced by the ridge push. This puts an upper bound on this shear stress of about 2×10^5 Pa. The evolution of the vertical and horizontal stresses away from the ridge is schematized on figure 4.

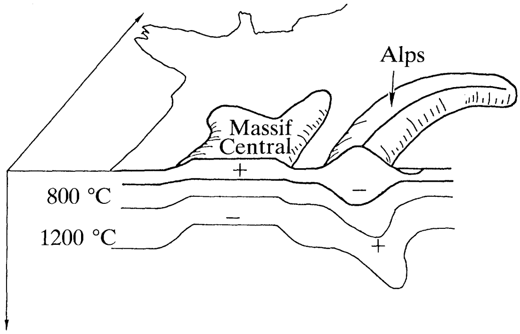

Figure 3. Schematic cross section through the Alps and the Massif Central. Crustal thickening implies lighter material at depth while the thickening of the cold (dense) mantle material forming the lower part of the lithosphere implies denser material at depth. As in figure 4, the deeper mass-anomaly has a predominant tectonic influence.

(b) *Ridge push compared with the other forces applied to the plates*

The slab pull can correspond to a very large force applied to the plates; it is of the order of 3×10^{13} Pa m. It is, however, locally counterbalanced by the colliding resistance and the resistance to penetration of the slab, so that the net force applied to the plate is not very large (Forsyth & Uyeda 1975). As argued by Bott, the fact that the state of stress is compressive in old oceanic lithospheres, close to subduction zones, even implies that the colliding resistance plus the viscous resistance to slab penetration exceed the slab pull. The sum of these forces corresponds to the deviatoric stress at old ages on figure 4.

Note that these considerations are only valid as averages and that there can be huge local variations of the force exerted by the slab depending upon the age of the subducting slab, the nature of the subducted material, the length of the slab, etc. These factors can induce large perturbations of the state of stress in the plates (Cloetingh & Wortel 1985).

(c) *The state of stress in old continental platforms*

During recent years seismic tomography has revealed thick lithospheric roots (reaching to about 400 km) beneath old continental platforms. Simple isostatic considerations imply that the seismic anomalies are not purely thermal but are also due to a petrological difference between the oceanic and continental mantles at similar depths (Jordan 1978). What about the variation of vertical stress due to such thermal and petrological features of the lithosphere? This will of course depend upon the relative distribution of the petrological and thermal anomalies as a function of depth. However, the geoid jump across passive margins is also proportional to the moment of the differential-density anomalies and thus can be used to estimate this moment. This geoid jump at passive margins with an old craton of moderate altitude amounts to about 5 m (it reaches 10 m for high continental platforms such as South Africa). This means that the moment of the density anomalies is about 10^{12} Pa m and that the averaged vertical stress is similar to that of an oceanic lithospheric about 50 Ma old (the latter is compatible with standard crustal densities, a linear temperature profile in the mantle varying between 450 °C at the base of the crust and 1300 °C at the base of the lithosphere and a petrological density-anomaly which is constant throughout the lithosphere and equal to $-0.026\ \mathrm{Mg\ m^{-3}}$). As 50 Ma old

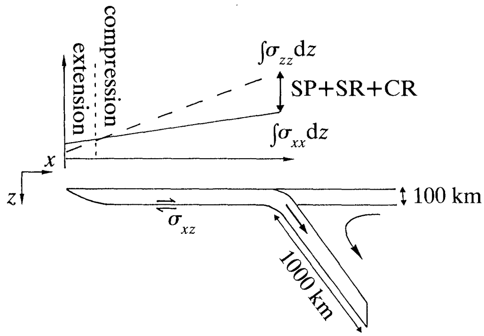

Figure 4. Simplified evolution of the horizontal and vertical stresses in an oceanic lithosphere pulled by a slab. The shear stress associated with the viscous drag beneath the plate implies an increase of the horizontal stress away from the ridge. As the lithosphere cools (i.e. becomes denser) away from the ridge, the vertical stress also increases. The 'tectonic stress' which corresponds to the difference between the horizontal and vertical stresses is observed to be extensional close to the ridge and compressive at older ages.

oceanic lithospheres are observed to be in compression, old continents are also expected to be in compression if the basal shear stress does not modify significantly the horizontal stress. Indeed most of the time they are observed to be in compression.

4. Deep density anomalies

The 'moment law' indicates that the deeper a density anomaly is in the lithosphere, the larger the tectonic stress that it induces. What then is the influence of the very deep density anomalies linked to the seismic anomalies detected during recent years by seismic tomography? The answer to this question depends upon the structure of mantle convection.

1. For one-layer mantle convection the degree 2 and 3 density anomalies, corresponding to the seismic anomalies found in the lower mantle and responsible for the low-harmonic geoid, should induce both huge tectonic stresses in the lithosphere (larger than 10^{13} Pa m) and huge topography anomalies (of the order of ∓ 1 km) (Ricard *et al.* 1984).

2. For two-layer mantle convection the lower-mantle mass anomalies induce a deflexion of the 670 km discontinuity. However, for appropriate viscosity contrasts between upper and lower mantle, the lithosphere is found to be mechanically decoupled from the lower mantle and neither vertical stress (corresponding to topography), nor shear stress (inducing tectonic stress) are induced at the base of the lithosphere by lower-mantle mass anomalies.

No topography anomaly is observed above lower-mantle mass anomalies. This result has been obtained by an inversion of oceanic topography as a function of age plus terms proportional to the geoid (Colin & Fleitout 1990). (A new inversion has been performed without flattening terms in the expression characterizing the topography as a function of age. We got the following result: $\text{topo} = 3026 + 246\sqrt{\text{age}} - 1.1\text{geoid}^2 - 3.35\text{geoid}^3 - 1.28\text{geoid}^4 \ldots$ This still indicates very little topography associated with the low-order harmonic geoid.) Observations of variations of altitude recorded by shallow marine deposits, as the plates drift above highs and lows of the geoid, confirm this conclusion. Therefore, we prefer a two-layer

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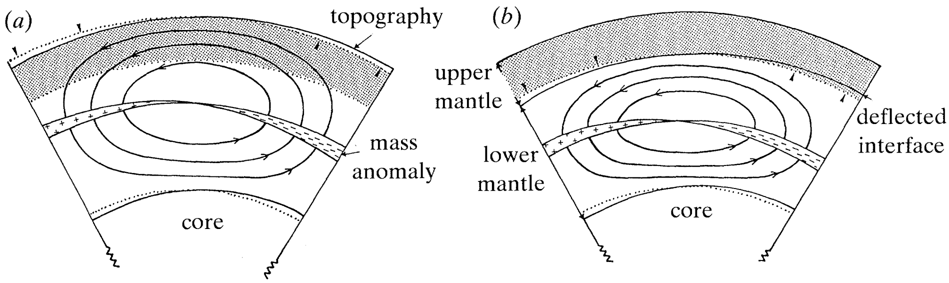

Figure 5. Influence of deep mantle density heterogeneities on the lithospheric stresses. For one-layer mantle convection (*a*), the lower-mantle mass heterogeneities induce huge lithospheric stresses and topography anomalies (of the order of 10^8 Pa × 100 km and ∓1 km respectively). For two-layer mantle convection (*b*), the lithosphere is mechanically decoupled from the lower mantle. The relief and the stress field are not affected by lower-mantle mass heterogeneities.

convecting mantle model where the lithosphere is mechanically decoupled from the lower mantle. Note that models where there is a convective layering of the mantle, but where the cold lithosphere can intermittently penetrate the lower mantle, would also fit these observations. In these two-layer or 'one-and-a-half-layer' mantle convection models, lower-mantle mass anomalies, which are the main source of the geoid, are decoupled from the lithosphere.

Upper-mantle seismic-tomography anomalies represent a complex mixture of chemical effects and thermal effects. It may be dangerous to interpret them simply in terms of density anomalies. Their tectonic effect should anyway be relatively minor, except for the variations in lithospheric structure discussed in §1.

5. Conclusion

What are the ingredients which must be included in a large-scale numerical model of the stresses in a plate? The lithospheric mass heterogeneities, among which are the ridge push and a resistive viscous drag below the plates, constitute the 'body forces' inside the plate. At the ridge the deviatoric stress seems to be very close to zero. The boundary condition at the trenches is more speculative. It is not yet well known how slab-pull, resistance to the slab penetration and collisional resistance vary with the geometry of the slab and the rate of subduction. Indeed, the stress field could be considered as a useful constraint on these quantities. As indicated in §4, deeper mass anomalies do not seem to play a major role in the stress field.

This paper only discussed the 'body forces' and the boundary conditions necessary for modelling and understanding the stress field. There is no doubt that the constitutive relationship relating internal deformation to the stresses is also important. In particular faults can considerably modify the stress pattern.

References

Artyushkov, E. V. 1973 Stresses in the lithosphere caused by crustal thickness inhomogeneities. *J. geophys. Res.* **78**, 7675–7708.

Babuska, V., Plomerava, J. & Sileny, J. 1984 Large scale orientated structures in the subcrustal lithosphere of Central Europe. *Annls Geophysicae* **16**, 117–123.

Brereton, R. & Müller, B. 1991 European stress: contributions from borehole breakouts. *Phil. Trans. R. Soc. Lond.* A **337**, 165–179. (This volume.)

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Cloetingh, S. A. P. L. & Wortel, M. J. R. 1985 Regional stress field of the Indian plate. *Geophys. Res. Lett.* **12**, 77–80.

Colin, P. & Fleitout, L. 1990 Topography of the ocean floor: thermal evolution of the lithosphere and interaction of deep mantle heterogeneities with the lithosphere. *Geophys. Res. Lett.* **17**, 1961–1964.

Dahlen, F. A. 1981 Isostasy and the ambient state of stress in the oceanic lithosphere. *J. geophys. Res.* **86**, 7801, 7807.

Fleitout, L. & Froidevaux, C. 1982 Tectonics and topography for a lithosphere containing density heterogeneities. *Tectonics* **1**, 21–56.

Fleitout, L. & Froidevaux, C. 1983 Tectonic stresses in the lithosphere. *Tectonics* **2**, 315–324.

Forsyth, D. & Uyeda, S. 1975 On the relative importance of the driving forces of plate motion. *Geophys. Jl R. astr. Soc.* **43**, 163–200.

Froidevaux, C. & Isacks, B. L. 1984 The mechanical state of the lithosphere in the Altiplano-Puna segment of the Andes. *Earth planet. Space Lett.* **71**, 305–314.

Jordan, T. H. 1978 Composition and development of the continental tectosphere. *Nature, Lond.* **274**, 544–548.

Ricard, Y., Fleitout, L. & Froidevaux, C. 1984 Geoid heights and lithospheric stresses for a dynamic Earth. *Annls Geophysicae* **12**, 267–286.

Stein, S. & Pelayo, A. 1991 Seismological constraints on stress in the oceanic lithosphere. *Phil. Trans. R. Soc. Lond.* A **337**, 53–72. (This volume.)

Discussion

P. England (*Oxford University, U.K.*) I would disagree that the continents are negatively buoyant because of deep cold roots because Dr Fleitout's results are very sensitively dependent on the assumed distribution of density with depth and, secondly, upon the assumption that lithosphere is preserved during the deformation. I would like to argue from her data that in fact the lithosphere is not preserved during compressional deformation. I noticed that the big negative velocity anomalies were concentrated under the Aegian, Turkey and the Caucasus which respectively are undergoing compression, strike–slip faulting and compression. So there does not seem to be a very strong correlation between upper-mantle thermal state and tectonic style. Secondly, Romanowicz and co-workers demonstrated that the thickness of lithosphere under Tibet is very much less than you would expect than if you had simply, homogeneously shortened the lithosphere layer by a factor of two. Although I would agree with her that undeformed continental lithosphere appears to be negatively buoyant with respect to the mid-ocean ridges it appears that once you start compressing that lithosphere it doesn't stay negatively buoyant for some reason.

N. Kusznir (*Liverpool University, U.K.*). If you want to get rid of the root by thermal assimilation you could explain why orogeny is going through extension but if you get delamination it falls off and you still have a negatively buoyant zone down there which should be subject to the same physics in terms of the density contrast moment. Although it may not be physically attached to the lithosphere above it should still have an effect. Indeed if one takes the density contrast moment into account the deeper it goes the greater the compression. One somehow senses one is missing something from the physics here.